Digital quantum batteries: Energy and information storage in nano vacuum tube arrays

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Abstract

Dielectric material between capacitor electrodes increases the capacitance. However, when the electric field exceeds a threshold, electric breakdown in the dielectric discharges the capacitor suddenly and the stored energy is lost. We show that nano vacuum tubes do not have this problem because (i) electric breakdown can be suppressed with quantization phenomena, and (ii) the capacitance is large at small gap sizes. We find that the energy density and power density in nano vacuum tubes are large compared to lithium batteries and electrochemical capacitors. The electric field in a nano vacuum tube can be sensed with MOSFETs in the insulating walls. Random access arrays of nano vacuum tubes with an energy gate, to charge the tube, and an information gate attached to the MOSFET, to sense the electric field in the tube, can be used to store both energy and information.

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Electrical energy from a DC power source can be stored in conventional capacitors, electrochemical capacitors, chemical batteries, and diodes. Conventional parallel plate capacitors can be charged and discharged quickly and they have a virtually unlimited life time, but their energy density is small, because of dielectric breakdown. For instance, when the electric field E exceeds about $0.118V/nm$ [1] in a $25\mu m$ thick Teflon sheet coated with $w = 100nm$ thin-film electrodes, a spark discharges the capacitor and the energy is lost as heat. This limits the energy density in aluminum-Teflon-aluminum capacitors to about $u = \epsilon_0\epsilon_r E^2/2 = 265kJ/m^3$ $(125J/kg)$, where $\epsilon_0 = 8.85 \times 10^{-12}F/m$ is the vacuum permittivity and $\epsilon_r = 2.15$ relative permittivity of Teflon. Commercial capacitors have energy densities up to $300J/kg$ [2](See Fig. 1). The inductance of the capacitor circuit limits the rate at which the capacitor can be charged and discharged to about $10^7W/kg$ [2].

In molecules and atoms, such as the hydrogen atom, and in solids, such as metallic Li, electric fields can be much larger because quantization phenomena suppress charge recombination. Consequently, the electrostatic energy density can be much higher than in conventional capacitors [3]. For example, the electric field at the surface of a hydrogen nucleus is about $10^{11}V/nm$ and when a hydrogen molecule is dissociated and ionized in water, two hydrated hydronium ions are created and the local increase in energy density is $(2 \times 13.6eV + 4.52eV)/4 \times 10^{-28}m^3 = 29GJ/m^3$ $(29MJ/kg)$. Therefore, devices which use electrochemical reactions and other faradic processes can store more energy than conventional capacitors. Commercial electrochemical capacitors can reach an energy density of up to $18kJ/kg$, pseudo capacitors reach up to $108kJ/kg$ and, in chemical batteries, the energy density can be as large as $540kJ/kg$ [2](See Fig. 1). However, faradic processes have small charge-discharge rates because they depend on the diffusion of ions in the electrolyte ($< 10^5W/kg$ for electrochemical capacitors, $< 10^3W/kg$ for batteries) and on chemical reaction rates (See Fig. 1). Furthermore, the number of charge/discharge cycles of electrochemical capacitors and batteries is small. Instabilities in the electro-crystallization lead to metal dendrites, that can cause short circuits, and irreversible chemical reactions can create films of solid material on the electrodes [2].

Solid state diodes, such as varactor diodes, can be used for energy storage as well. Semiconductor diodes with reverse bias store energy [3] in the depletion layer. However, field emission, avalanche breakdown and Zener breakdown limit the electric field to about $E = 0.02V/nm$ (in silicon with donor concentration $N = 10^{14}cm^{-3}$) [4] and the energy density

is less than in conventional capacitors. Field emission, i.e. quantum mechanical tunneling of carriers through the band gap, is the dominant breakdown mechanism for highly doped p-n junctions. Zener breakdown occurs when the electric field becomes large enough to excite valence electrons in the depletion zone directly into the conduction band. Avalanche breakdown is when the minority carriers are accelerated in the electric field in the depletion region to sufficient energies that they can excite valence electrons through collisions. Energy storage in semiconductor junctions is further limited by the fact that the depletion zone is not a perfect insulator and reverse saturation currents discharge the diode. The charge-discharge rate of diodes is limited by the mobility of the carriers, much higher than in batteries. Tunnel junction have the highest switching speeds (up to 5GHz), but their reverse-biased breakdown voltage is small.

Nano plasma tube arrays with nano-tip electrodes and flat electrodes conduct current only if the tip is charged negatively (forward direction) and if the electric field at the tip exceeds a threshold. If the electric field is reversed there is no current. Only at a much higher electric field current will start to flow in reverse direction. Therefore nano plasma tubes can be used as diodes and triodes [5], and in reversed direction, they can be used for energy storage like a varacter diode or a capacitor. Nano plasma tubes are generally forward-biased and if the residual gas emits visible light, they can be used for flat-panel plasma lamps and flat panel monitors [6]. Their switching speed in the THz range[6], which is a factor of 1000 larger than the fastest semiconductor junctions. One of the original two device types, the "Spindt array" [7], used silicon-integrated-circuit fabrication techniques to make regular arrays in which molybdenum cones were deposited in small cylindrical voids in an oxide film, with the void covered by a counter electrode with a central circular aperture. The energy density density in reverse-biased nano plasma tubes is small, because gas becomes a partically ionized, conducting plasma at comparatively small electric fields.

In this paper, we investitage energy storage in arrays of reverse-biased nano vacuum tubes, which are similar in design to nano plasma tubes, but contain little or no gas. The key design parameter is the gap size, the distance between the electrodes. Electrical breakdown in vacuum gaps has been studied or more that 80 years [8–10] for gap sizes above 200nm. However little is known about vacuum gaps in the nanometer range. We show that in reverse bias, the electric field near nano-tip anodes can be orders of magnitudes larger than breakdown field in conventional capacitors, varactor diodes, and nano plasma tubes.

Since there are only residual gases between the electrodes in vacuum junctions, there is no Zener breakdown, no avalanche breakdown, and no material that could be ionized. Electrical breakdown is triggered by quantum mechanical tunneling of electrode material: electron field emission on the cathode and ion field emission on the anode. Because the energy barrier for electron field emission is large and the barrier for ion field emission even larger, the average energy density in reversed-biased nano vacuum tubes can exceed the energy density in solid state tunnel junctions and electrolytic capacitors. Since the inductance of the tubes is very small, the charge-discharge rates exceed batteries and conventional capacitors by orders of magnitude. Charging and discharging involves no faradaic reactions so the lifetime of nano vacuum tubes is virtually unlimited. The volumetric energy density is independent from the materials used as long as they can sustain the mechanical load, the electrodes are good conductors, and the mechanical supports are good insulators. Therefore, nano vacuum tubes can be built from environmentally friendly, non-noxious materials. Materials with a low density are preferable, since the gravimetric density is the ratio between the volumetric energy density and the average density of the electrodes and supports. Leakage currents are small, since the residual gases contain very few charged particles. Nano vacuum tubes can be fabricated with standard photo lithographic techniques [11] and could be easily integrated in integrated circuits as a rechargeable battery.

We consider nano vacuum tubes with planar cathodes. The anodes are sharply pointed tips, such as Spindt tips [7, 12] or the endings of cantilever nanotubes [13]. Figure 2 shows a sketch of a four nano vacuum tubes. The tips are arranged in a square grid at distance s. The gap size d is the shortest distance between the surface of the tip and the surface of the cathode. The height of the tube h is roughly equal to the apex radius of the tip r plus the gap size d, *-i.e.* $h \approx r + d$ and the distance between the tips is equal to the gap size plus the width of the insulating mechanical supports w, -i.e. $s \approx d + w$. The thickness of the planar electrodes is t. The smallest tips are single-atom-apex Mueller emitters with apex radius $r \approx 0.1nm$, about twice the crystallographic size of a single atom [14]. Techniques for preparing them have been under investigation for many years [15, 16]. Recently a automatic technique has been developed for restoring a single-atom apex to its original state, if the top atom moves away from this position [14]. Another type of tip-anode are free standing nano-wires, with an apex radius of $r \approx 5nm$, clamped in normal direction to a planar metal anode. Pre-breakdown heating of the anode and the anode current will generally be small

because the electrons penetrate several layers of the anode material and spread the heat over a large volume.

The energy density of the tubes is limited by vacuum breakdown. At gas pressures of less than $10^{-6}Torr$ the breakdown field does not depend on the residual gas, but on the properties of the electrode surfaces [10].

The local cathode fields, enhanced by a a factor β by microprotusions and other surface defects[17], determines the breakdown field $E_b = \beta E_s$, where E_s is the macroscopic field on the cathode surface when an arc develops. Figure 3 shows the relation between the field enhancement factor β and the gap size d. The function

$$\beta = \left(\frac{d}{0.4nm}\right)^{1/3} \tag{1}$$

approximates the relation between β and the gap size d for tungsten electrodes for $d > 1nm$ [9, 10]. The field enhancement factor of copper electrodes, niobium electrodes, or niobium electrodes with a layer of gold has a similar dependence on the gap size [18]. However β is larger for thinner and taller cathodes [19] and β tends to get smaller if the the electrodes are thermally treated or allowed to break down many times. We consider only tubes with wide and short cathodes. The limiting value for *beta* is equal to one for gap sizes of the size of one atom (see Fig. 3).

The largest component of cathode current is Fowler-Nordheim field emission [8]. Fowler-Nordheim field emission is a quantum mechanical phenomenon, which describes the tunneling of electrons through the energy barrier at a charged metal-vacuum interface. The current density J depends on the electric field E at the surface, $J = J_0(E^2/E_0^2)e^{E_0/E}$, where $E_0 = f\Phi^{3/2}$ and Φ is the work function of the metal. J_0 and f are material constants. For niobium the work function is at 4eV and $J_0 = 1.15 \times 10^{15}A/m^2 = 1.15kA/\mu m^2$ and $E_0 = 54.6V/nm$ [18]. Breakdown occurs roughly at about twice the onset of field emission, i.e. $E_0 \approx E_b$. The breakdown field has a very small temperature dependence, -i.e. the breakdown field is almost same at room temperature and 100^oC [18]. Clean cathodes of tungsten, copper, niobium, or niobium with a layer of gold have a breakdown field of about $E_b = 60V/nm$ [10, 18]. The maximum energy density near the cathode, $u_c = \frac{\epsilon_0}{2}E_s^2$ depends on the gap size d (see Eq. 1):

$$u_c = \frac{\epsilon_0}{2}\frac{E_b^2}{\beta^2} = \epsilon_0 E_b^2 \left(\frac{0.4nm}{d}\right)^{2/3} \tag{2}$$

where $\epsilon_0 = 8.85 \times 10^{-12} \frac{F}{m}$ is the vacuum permittivity. Fig. 4 shows the maximum energy density near the cathode as a function of the gap size. The maximum energy density is larger for smaller gap sizes. For example, in a tube with gap size $d = 25nm$ the maximum energy density at the cathode is a factor of 100 larger than in a tube with gap size $d = 25\mu m$. The electric field creates a force on the surface of the cathode. The pressure p_c on the cathode surface due to electric fields equals the energy density at the surface, i.e. $p_c = u_c$. The cathode disintegrates if u_c exceeds the tensile strength P_t of the cathode material. Therefore the tensile strength of the cathode material limits the energy density near the cathode u_c too. Figure 4a illustrates the tensile strength of typical cathode materials. The shaded area in Fig. 4b indicates where the energy storage is stable in a tube with a tungsten cathode. If the voltage on a tube with gap size $d = 10nm$ is increased, the energy density at the cathode increases, until it reaches $u_c = 1.51GJ/m^3$. The pressure on the electrode then exceeds the tensile strength of tungsten and the electrode disintegrates. However, if a vacuum junction with gap size $d = 10\mu m$ is charged, field emission will trigger electrical breakdown at an energy density which is much below the tensile strength of tungsten (see Fig. 4b).

The mechanical supports and the electrodes form vacuum micro-tubes. Hwang et al. [11] have shown that is possible to fabricate nano vacuum tubes which are stable under atmospheric pressure. In the following we assume that the micro-tubes are roughly cubical, and that the tips are far enough from the planar anode that the electric field E in the tube can be approximated by a point charge with its image charge at the cathode. Furthermore, we ignore the interaction between the tubes. We introduce a coordinate system where the apex centers of the tips are in the $x - y$ plane and the origin is at the apex center of one of the tips. The electric field at a position $\mathbf{x}$ in the tube between the electrodes is complicated [20] but can be roughly approximated by a point charge in front of a conducting plane

$$\mathbf{E}(\mathbf{x}) = E_a r^2 \left(\frac{\mathbf{x}}{|\mathbf{x}|^3} + \frac{\mathbf{x}_i - \mathbf{x}}{|\mathbf{x}_i - \mathbf{x}|^3} \right) \tag{3}$$

where E_a is the magnitude of the electric field at the surface of the anode and $\mathbf{x}_i = (0, 2d + 2r, 0)$ is the location of the image charge. We assume the largest electric field at the cathode is equal to the breakdown field, i.e. $|\mathbf{E}(0, d + r, 0)| = E_s$, and obtain with Eq. 1 and Eq. 3 a relation between the electric field at the anode and the electric field at the cathode.

$$E_s = \frac{E_b}{\beta} = 2E_a \frac{r^2}{(d + r)^2} \tag{4}$$

Therefore, the maximum energy density at the anode depends on the maximum energy density at the cathode u_c and the sharpness of the tip

$$u_a = \frac{\epsilon_0}{2} E_a^2 = u_c \frac{(1+S)^4}{4} = \epsilon_0 E_b^2 \left(\frac{0.4nm}{d}\right)^{2/3} \frac{(1+S)^4}{4} \tag{5}$$

where the ratio $S = \frac{d}{r}$ is a measure for the sharpness of the tip. If the energy density at the anode is larger than this value, then field emission at the cathode triggers an electrical breakdown. A sharply tipped anode can have a very high energy density without exceeding the limiting energy density at the cathode. Figure 5 shows the maximum energy density at the anode versus the sharpness of the tip for gap sizes $d = 5nm$, $d = 200nm$ and $d = 1\mu m$ and apex radius $1nm < r < 10\mu m$. Figure 5a illustrates the tensile strength of typical anode materials. The shaded area in Fig. 5b indicates where the energy storage is stable in a tube with a carbon nanotube anode and a cathode that does not fail mechanically. If the voltage on the tube with apex radius $r = 10nm$ is increased, the energy density at the anode increases, until it reaches $u_a = 62GJ/m^3$. The pressure on the electrode then exceeds the tensile strength of carbon nanotubes and the anode disintegrates. However, if a vacuum junction with apex $r = 10\mu m$ is charged, field emission on the cathode will trigger electric break down even if energy density anode is much below the tensile strength of carbon nanotube anode (see Fig. 4b). With Eq. 1 - 4 we conclude that the average electric field at the cathode E_s is limited by (i) the field enhancement factor β, which is a function of the gap size d, (ii) the work function E_b of the cathode material, (iii) the tensile strength of the cathode P_c and the anode P_a and (iv) the sharpness of the anode S:

$$E_s = E_a \frac{2r^2}{(d+r)^2} = \min\left\{\frac{E_b}{\beta}, \sqrt{\frac{2P_c}{\epsilon_0}}, \sqrt{\frac{2P_a}{\epsilon_0(1+S)^4}}\right\} \tag{6}$$

Within the vacuum tube, the electric field is largest near the y-axis within the region $R = \left\{(x,y,z)|x^2 < \frac{d^2}{4(d+r)^2}y^2 \text{ and } z^2 < \frac{d^2}{4(d+r)^2}y^2\right\}$. Therefore average energy density u in the tube at breakdown is estimated from the electric field near the y-axis

$$\begin{aligned} u &\approx \frac{\int_r^{r+d} \frac{\epsilon_0}{2}|\mathbf{E}(0,y,0)|^2 \left(\frac{d^2}{(d+r)^2}y^2\right) dy}{d^3} \\ &= 2\epsilon_0 E_a^2 \frac{\left(3 + 12\frac{d}{r} + 16\left(\frac{d}{r}\right)^2 + 6\left(\frac{d}{r}\right)^3\right)}{3\left(1 + 3\frac{d}{r} + 2\left(\frac{d}{r}\right)^2\right)^3} \end{aligned} \tag{7}$$

With Eq. 4 field emission at the cathode triggers electric breakdown if the average energy density exceeds

$$u = \frac{\epsilon_0}{2} E_s^2 \frac{(1+S)\left(3+12S+16S^2+6S^3\right)}{3\left(1+2S\right)^3} \approx \frac{\epsilon_0}{2} E_s^2 \begin{cases} 1 & \text{if} \quad 0 \le S \le 2 \\ \frac{S+2}{4} & \text{if} \quad S > 2 \end{cases} \tag{8}$$

where $S = \frac{d}{r}$ is the sharpness of the anode. Therefore

$$u = e_t \cdot \min\left\{ \frac{\epsilon_0}{2} \frac{E_b^2}{\beta^2}, P_c, \frac{P_a}{(1+S)^4} \right\} \tag{9}$$

where e_t is the tip enhancement factor, and $e_t = 1$ for $0 \le S \le 2$, $e_t = (S+2)/4$ for $S \ge 2$, and $e_t = S/4$ for $S \gg 2$ In small tubes, the average energy is often limited by the tensile strength of the electrodes (P_c and P_a), and not by electric breakdown. For instance, in a tube with a flat tungsten electrodes and gap size $d = 40nm$, the average energy density is limited by the tensile strength of the tungsten electrodes (see Fig. 4b). The limiting value of the average energy density is $u = P_t = 1.51 \times 10^9 J/m^3$, which is about a factor of 10^4 larger than in conventional capacitors. Carbon nanotubes have a very high tensile strength of about $62GPa$ ($300GPa$ theoretical limit). Therefore we propose to use carbon nanotubes for the anode and tungsten for the cathode. For a tube with a carbon nanotube anode ($r = 5nm$), a flat tungsten cathode and a the same gap size $d = 40nm$ the average energy density is $u = 2.5P_t = 3.78GJ/m^3$.

The wall material, such as SiO_2, needs to support this load. According to Pascal's law, the minimum cross sectional area of the wall material A_w is:

$$A_w = A_c \frac{u}{e_t P_w} \tag{10}$$

where P_w is the compressive strength of the wall material and A_c is the area of the cathode. The compressive strength of SiO_2 is about $P_w = 10^9 Pa$. For a tube with flat electrodes ($S = 0$) and gap size $d = 40nm$ and $u = 1.51GJ/m^3$, the cross sectional area of the wall has to be at least 1.51 times the area of the cathode, i.e. $A_w/d^2 = 1.5$. In this case the average density of the tube is not much less than the density of the wall material. If we assume that electrodes are very thin, -*i.e.* $t \ll d$, the average density of the tube is $\rho = \frac{A_w \rho_w + A_c \rho_g}{A_w + A_c}$, where ρ_g is the density of the residual gas in the tube. Since the density of the residual gas is very small $\rho_g \ll \rho_w$, the density can be approximated by $\rho = \frac{A_w \rho_w}{A_w + A_c}$. Then the gravimetric energy density u_g at breakdown is:

$$u_g = \frac{u}{\rho} = \frac{u}{\rho_w} + u_{g,min} \tag{11}$$

where $u_{g,min} = e_t \frac{P_w}{\rho_w}$ is the minimum gravimetric energy density. If the average energy density u is small, the walls can be very thin, and therefore the gravimetric energy density is at least $u_{g,min}$. Therefore the gravimetric energy density is in the range:

$$e_t \frac{P_w}{\rho_w} \leq u_g \leq \frac{u}{\rho_w} + e_t \frac{P_w}{\rho_w} \tag{12}$$

where $e_t \geq 1$ is the tip enhancement factor. If the energy density is less than the compressive strength of the wall material, then the insulating walls can be thin, and thus the compressive strength of the walls sets a lower limit for the gravimetric energy density. For SiO_2 walls the ratio $\frac{P_w}{\rho_w} = 200kJ/kg$ is only a factor of two less tan the maximum gravimetric density of chemical batteries. For a tube with flat tungsten electrodes and gap distance $d = 40nm$, the gravimetric energy density is $u_g = 600kJ/kg$. For a tube with a carbon nanotube anode ($r = 5nm$), a flat tungsten cathode, SiO_2 walls, and a the same gap size $d = 40nm$, the gravimetric energy density is $u_g = 1.5MJ/kg$. This value is a factor of three larger than the maximum gravimetric energy density of Li-metal batteries (See Fig. 1).

The most significant drawback with nano vacuum tubes is the high voltage. The largest potential difference between the electrodes is $\Delta V = \int_r^{r+d} \mathbf{E}(0, y, 0) dy = E_a r^2 \frac{d}{r(d+r)}$. With Eq. 4 and Eq. 1 we obtain

$$\Delta V = E_s d \cdot e_v \tag{13}$$

where E_s is the electric field at the cathode, and $e_v = \frac{(1+S)^2}{1+2S}$ is the voltage enhancement factor. The voltage enhancement factor can be approximated by $e_v = 1$ if $0 \leq S \leq 2$ and $e_v = S/2$ else. For a tube with flat electrodes ($S = 0$) with gap size $d = 40nm$ and $E_b = 60V/nm$, the potential difference between the electrodes is $\Delta V = 24V$. For a tube with a carbon nanotube anode ($r = 5nm$), a flat tungsten cathode, SiO_2 walls, and gap size $d = 40nm$, the voltage is $\Delta V = 114V$.

The volumetric capacitance $c = \frac{2u}{\Delta V^2}$ of nano vacuum tubes is inverse proportional to the square of the gap size:

$$c = \frac{\epsilon_0}{d^2} \cdot \frac{e_t}{e_v} \tag{14}$$

Therefore tubes with a smaller gap size have a higher capacitance. If the anode is flat ($S = 0$), then the capacitance is $c = \frac{\epsilon_0}{d^2}$ and tubes with sharply pointed anodes ($S \geq 2$) the the capacitance is approximately $c \approx \frac{\epsilon_0}{d^2} \cdot \frac{2}{S}$. This means tubes with sharply pointed anodes have a smaller capacitance. They can reach higher energy densities, but require

higher voltages to reach these energy densities. The gravimetric capacitance $c_g = \frac{2u_g}{\Delta V^2}$. For example, a tube with flat electrodes and a gap size $d = 40nm$ has a capacitance of $c_g = 2083F/kg$.

The charging-discharging rate of nano vacuum tubes is limited by their inductance, $f \approx \sqrt{\frac{c_0^2}{A_c}} = \frac{c_0}{d}$, where $c_0 = 3 \times 10^8 m/s$ is the speed of light and $A_c = d^2$ is the cathode area. Leak currents limit the charging-discharging rates too. The smallest discharging/charging rate is $f = \frac{1}{RC}$, where the resistance $R = \frac{A_c}{\sigma_w d}$ and the capacity $C = cA_c d$. Therefore the charging/discharging rates of nano vacuum tubes are in the rage

$$\frac{\epsilon_0 A_c}{\sigma_w A_w} \cdot \frac{e_t}{e_v} < f < \frac{c_0}{d} \tag{15}$$

For a cavitiy with flat electrodes, gap size $d = 40nm$ and SiO_2 walls ($\sigma = 10^{-11} \frac{1}{\Omega m}$) the charge-discharge rate is in the range $10^{-22} Hz < f < 7.5THz$. The lower limit is very small. This means that the tube holds the charge for many of years and thus can be discharged very slowly without any losses. The upper limit is very large (See Fig. 1). This means that the vacuum nano tube can be discharged very quickly and has a high power density. The maximum power density in a $40nm$ tube is $2.8 \times 10^{22} W/m^3$ ($1.1 \times 10^{19} W/kg$).

In conclusion, vacuum breakdown triggered by field emission limits the energy density in vacuum tubes. However smaller tubes have a smaller field enhancement factor β. Therefore, the energy density in smaller tubes is larger. The energy density can be further enhanced by using a nano-tip anode. The tip enhancement factor for the energy density $e_t = (S + 2)/4$, however the voltage difference is enhanced too, by a factor $e_v = \frac{(1+S)^2}{1+2S}$. Since nano vacuum tubes are empty, their average density is less than the density of the materials that constitutes them. This sets a lower limit for the gravimetric energy density $u_{g,min} = e_t P_w / \rho_w$, where P_w is the compressive strength of the wall material and ρ_w is the density of the wall material. Nano vacuum tubes with an energy density u of more than $1GJ/m^3$ have to be thick-walled. $u_{g,min}$ and the enhancement factors combined make nano vacuum tubes the devices with the highest gravimetric energy density (See Fig. 1). $u_{g,min}$ and the enhancement factors are due to the geometry of the device, and largely independent of the material properties, except for Eq. 1, the electric field E_b at breakdown and the compressive strength of the wall material. However Eq. 1 appears to hold for a large range of electrode materials and the breakdown field is pretty similar for many metals. Therefore the device should have a very similar energy storage capacity for a large range of electrode materials. For that reason,

cheap environmentally friendly materials can be used to fabricate it, including glass (SiO_2) as a wall material. The charging-discharging rates of nano tubes exceed all other devices (See Fig. 1). Vacuum nano tubes can hold electric energy without any losses for many years, and can be charged and discharged rapidly. The largest charging-discharging rate is proportional to ratio between the gap size and the speed of light (see Eq. 15), whereas the charging-discharging rate of batteries and electro-chemical capacitors depends on diffusion rates and chemical reaction rates. Conventional capacitors have a large inductance; therefore, their charging-discharging rate is comparatively small. The gravimetric power density of nano vacuum tubes could even exceed that of nuclear chain reactions because the gravimetric power density of nuclear chain reactions depends on the speed of neutrons which is much less than the speed of light. Since nano vacuum tubes use no faradic processes their life time is much longer than electrolytic capacitors and batteries. Nano vacuum tube arrays have great potential as power sources for electronic devices because they can be produced with standard lithographic processes and can be part of integrated circuits. Integrated circuits with an internal power source have small Ohmic losses compared to circuits with an external energy supply and produce less heat. If each cavitity is individually addressable, such as random access memory, the tubes can be charged and discharged in any arbritary order. If one inserts a MOSFET in the wall of the vacuum tube, the state of the tube can be determined without charging or discharging it. In this case each nano vacuum tube has two gates, an energy gate and an information gate, similar to the floating gate and control gates in flash drives. Vacuum nano tubes with both gates can be used for information and energy storage. For example to store the number 22 one would convert it binary notation $22 = 10110 < sub > 2 < /sub >$ and then use the energy gates to charge the first, third and fouth tube and leave the second and fifth tube uncharged. When the energy gate holds a charge, it induces an electric field in the MOSFET that partially cancels the electric field from the electrodes of the information gate, which modifies the threshold voltage of the MOSFET. During read-out, a voltage slightly above the regular threshold voltages is applied to the information gate, and the MOSFET channel will become conducting or remain insulating, depending on the voltage threshold of the MOSFET, which depends on the charge on the energy gate. The current flow through the MOSFET channel is measured and provides a binary code, reproducing the stored data.

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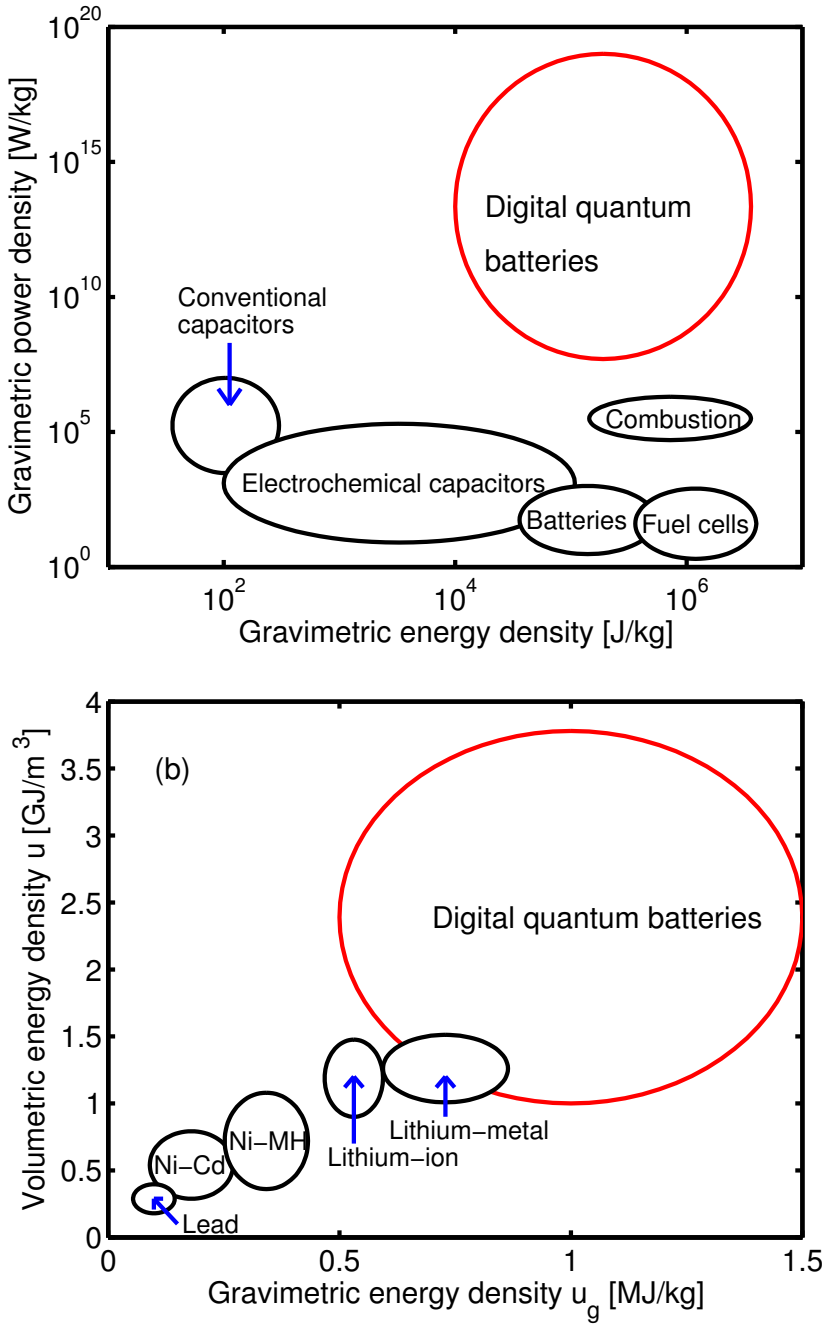

FIG. 1: The energy density and the power density of nano vacuum tubes in comparison to other energy storage devices (a). The volumetric energy density and the gravimetric energy density of nano vacuum tubes in comparison with combustion engines and several types rechargeable batteries, including those based on lead, nickel-cadmium, nickel-metal hydride, and lithium (b). Nano vacuum tube arrays are lighter and smaller energy storage devices than other batteries.

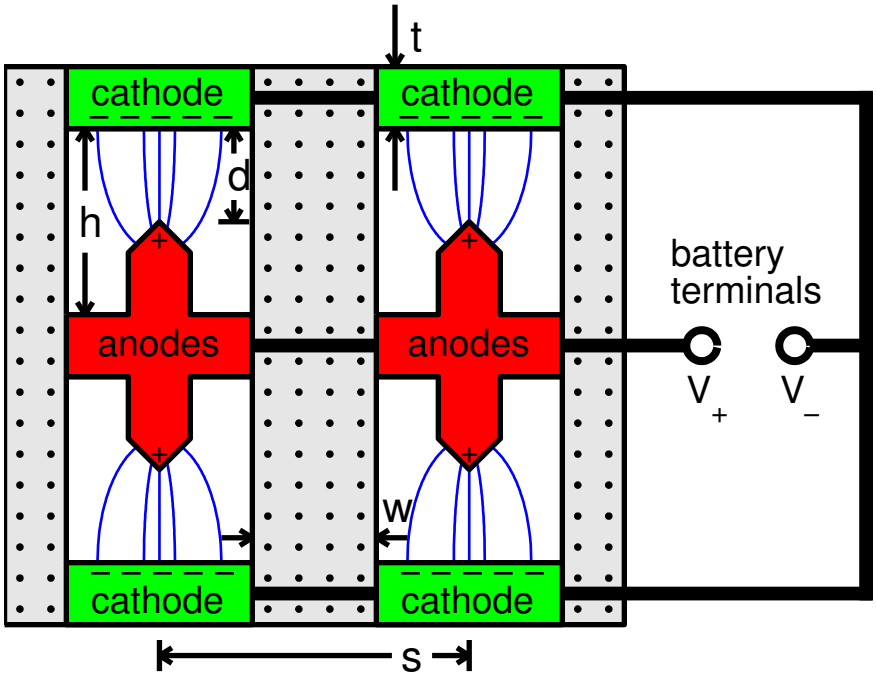

FIG. 2: Schematic of an array of four vacuum nano tubes (cross section, side view). The cathode $(---)$ is a planar. The anode $(+++)$ is a nano tip on a flat electrode. The thin curved lines indicate the electric field lines. The cathode is a conducting solid with high tensile strength, such as tungsten or steel. The flat part of the anode needs to have a high tensile strength, such as steel. The nano tip is a conducting solid with extremely high tensile strength, such as a carbon nanotube clamped to the steel electrode, or a tungsten Spindt-tip [7]. The design of the electrodes is similar to a tunneling microscope, except that tunneling microscopes have one movable tip, whereas the vacuum tube arrays have many stationary tips. The insulating walls (dots) are solids with a high compressive strength, such as silicon oxides. The electrodes and the walls create a vacuum tube. The electric field is in the vacuum tube between the anode tip and the cathode.

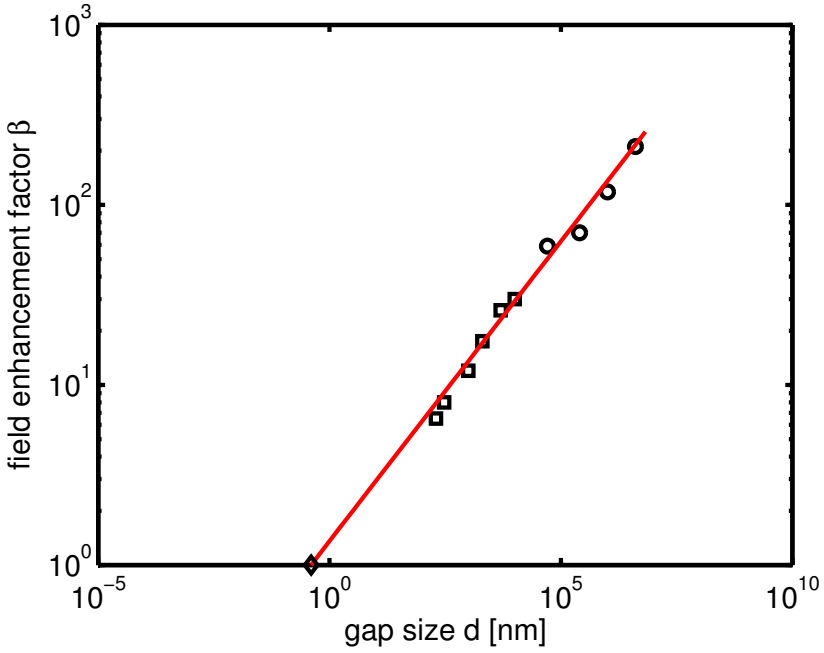

FIG. 3: The field enhancement factor β versus the gap size d. The circles indicate experimental values measured by Alpert, Lee, Lyman, and Tomaschke [10] on tungsten electrodes. The squares indicate experimental values measured by Boyle, Kisliuk, and Germer [9] on tungsten electrodes. The diamond indicates the theoretical value on an atomic scale. The continuous line is a fit of the experimental data (Eq. 1).

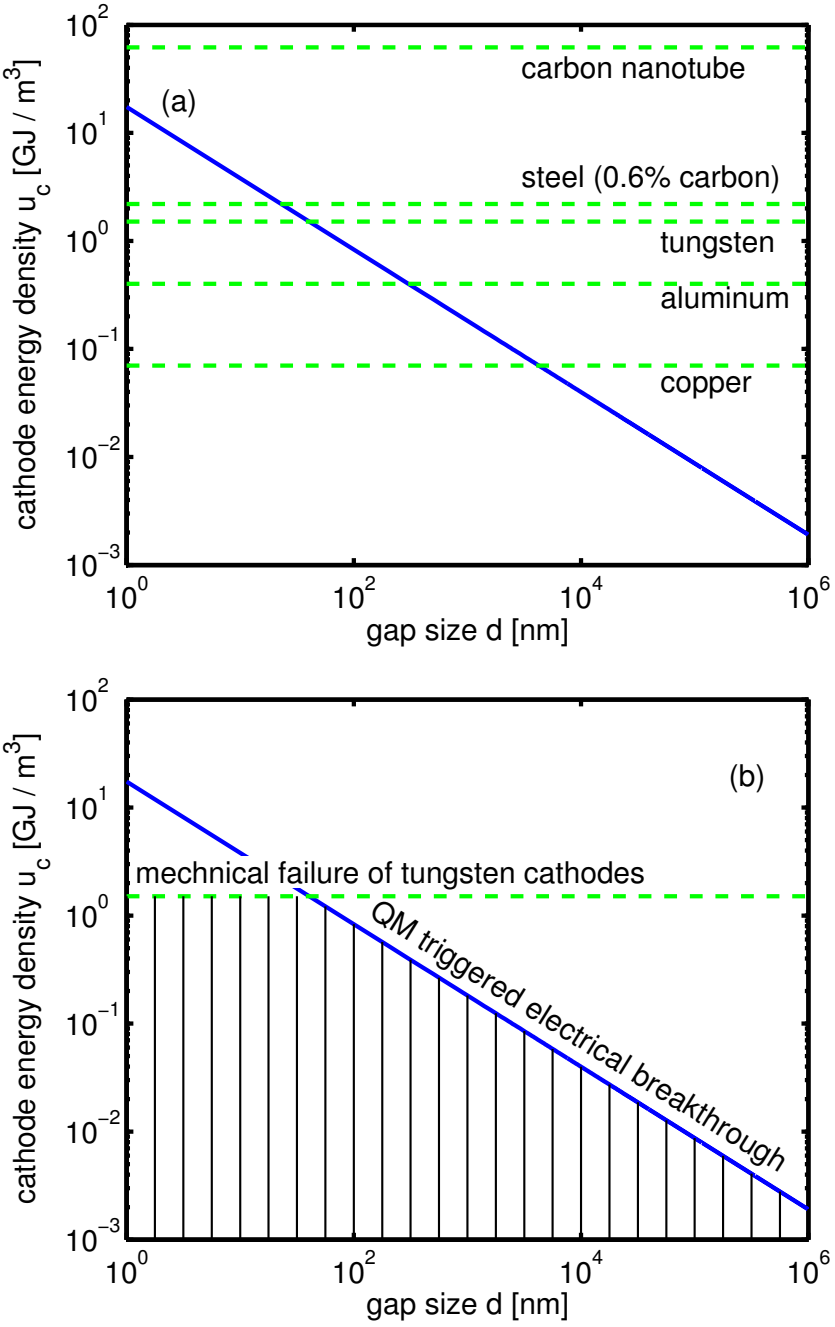

FIG. 4: Quantum mechanical tunneling triggers electric breakdown at high energy densities. The continuous line illustrates the energy density at the cathode at breakdown versus the gap size d in the range $1nm < d < 1mm$. Smaller tubes have a larger maximum energy density at the cathode. If the energy density exceeds the tensile strength of the electrode material the cathode breaks apart. The dashed line indicates the tensile strength of typical cathode materials (a). The shaded area indicates where a charged tungsten cathode is electrically and mechanically stable (b).

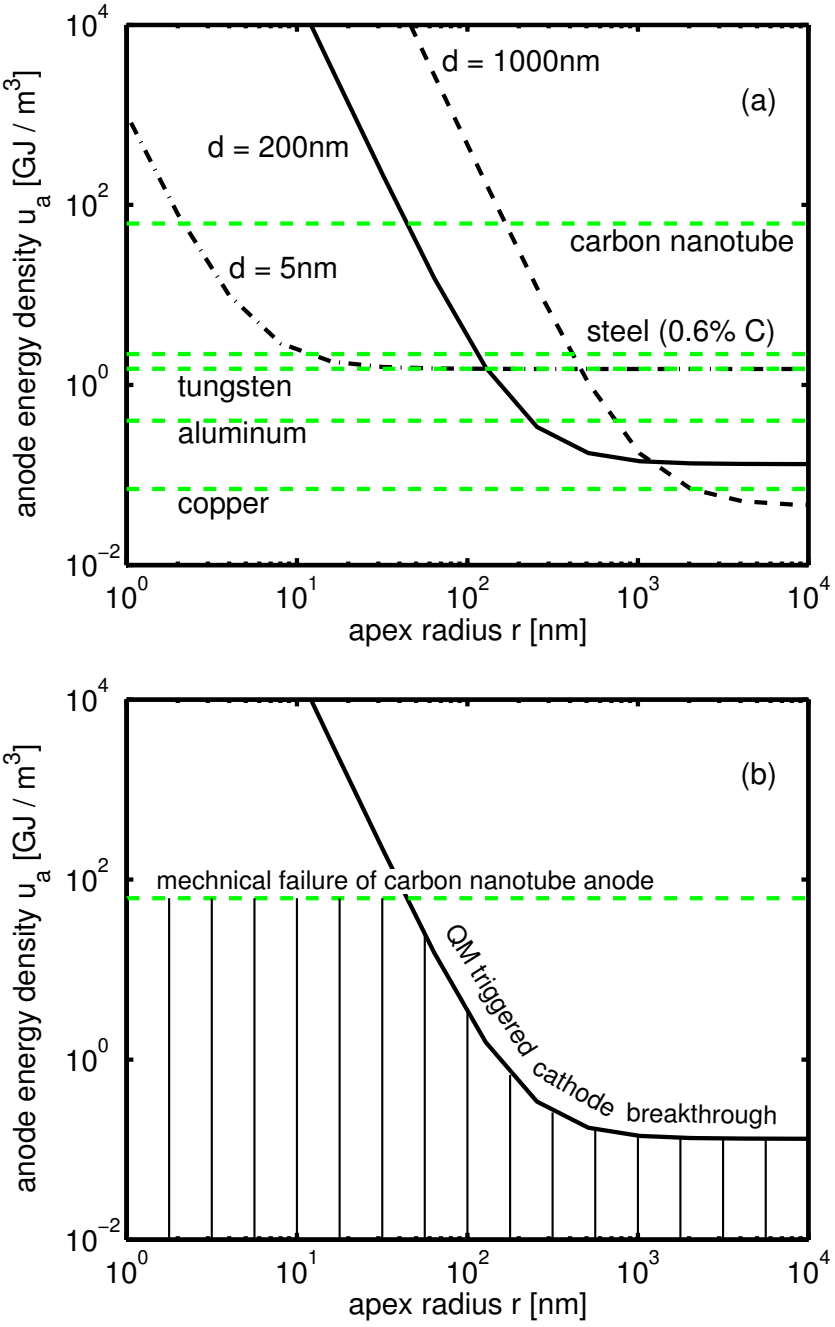

FIG. 5: The maximum energy density at the anode versus the apex radius r for three gap sizes d. Tips with a small apex radius have a larger maximum energy density at the anode. The dashed lines indicate the tensile strength of typical anode materials. The tensile strength of the anode material limits the energy density (a). The shaded region indicates where a nano vacuum tube with a carbon nanotube anode is mechanically and electrically stable (b).